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ABSTRACT

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Using Composite Variables in Multivariate Analysis:

Why Weight?

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ABSTRACT

Multivariate analysis techniques employ various methods of optimally weighting variables to form composites. The interpretation of these composites in subsequent analyses is rarely straightforward, fraught with difficulties based upon the statistical properties of the estimated weights. This paper synthesizes interpretive guidelines for four commonly used multivariate techniques: multiple linear regression, canonical correlation, MANOVA and linear discriminant analysis. Three methods of weighting variables are discussed and illustrated on simulated data. The complexity of the correlation structure of the entire set is shown to be the crucial determinant in selecting the weighting method of choice. Guidelines for the use of the various weighting schemes on all four multivariate techniques are presented.

The consideration of multiple sources of information in decision making, in evaluating the effects of a treatment, in studying group differences, or in predicting future outcomes is becoming standard practice in the behaviorial sciences. In many areas of education and psychology, for example, the use of multivariate research designs and statistical analysis methods is increasing. Glass (1976), in his presidential address to the American Educational Research Association, called for the increased usage of multivariate techniques in educational research to better understand the complex interrelationships and joint outcomes among variables operating in the educational process. Harris (1979), in an invited address to the American Psychological Association entitled "Multivariate Statistics: When Will Experimental Psychology Catch Up?", voiced a similar advocacy.

The exposition of multivariate techniques has evolved from solely sophisticated mathematical treatments (e.g., Anderson, 1958; Draper & Smith, 1966), which emphasized the calculus of maximization and minimization functions, to those which dealt with these procedures via verbal description and conceptual overview (e.g., Harris, 1975; Tatsuoka, 1971), and also those which illustrated their meaningful application on "real" data without delving too deeply into theoretical foundations (.e.g, Cooley & Lohnes, 1971). Thus, depending upon the user's orientation and interest, there now exists an appealing variety of treatments of the complexities of multivariate analysis.

The methods themselves all have at least one fundamental characteristic in common. They all seek to form linear combinations of the multiple variables which will maximize or minimize some established criterion. Multiple linear regression analysis, for example, solves for that linear combination of predictor variables which minimizes the sum of squared errors of prediction (or, equivalently, maximizes the squared multiple correlation between predictors and criterion). Linear discriminant analysis solves for that linear combination of dependent variables which maximizes the differences among the treatment groups under study. Similar statements can be made for other multivariate techniques (.e.g, factor analysis, MANOVA, canonical correlation analysis, etc.).

A key analytical step in these multivariate procedures is the derivation of the weights to be applied to the variables to form the optimal composite. Through the calculus of maximization or minimization, a set of weights, b_i, are derived to form a composite variable C asshown below:

$$C = b_1X_1 + b_2X_2 \ \ldots + b_pX_p \qquad (1)$$

The instability of these weights across samples and variable combinations is well documented in methodological textbooks (e.g., Kerlinger & Pedhazur, 1973) and journal literature (e.g., Darlington, 1968). Recent literature has questioned the need for such sophisticated calculation in light of the sample-specific interpretation in which the user is typically constrained. Wainer (1976) has shown algebraically that, in most instances, "it don't make no nevermind" in multiple linear regression whether least square weights or simple unit weights (-1,0,+1) are used in forming the composite. Schmidt (1971) has demonstrated this

to be accurate from a large sample of studies drawn from the literature in differential psychology. Einhorn and Hogarth (1975) add the field of industrial and organizational psychology as one which effectively applies unit weighting schemes in decision making. Dawes and Corrigan (1974) have evidenced the same conclusion in synthesizing a statistical and clinical weighting process. They conclude that, in forming composites of multiple variables, "the real trick is to know what variables to look at and then know how to add. (p. 105)"

In reading these and other articles like them, the researcher is tempted (perhaps happily) to discard the volumes of stepwise multiple regression references and never again look at a computer program designed to perform such analyses. Surely a set of relevant predictors can be selected, added up to form a composite, and a nearby calculator summoned to compute a simple Pearson correlation. Why bother with all of the confusion of multicollinearity, shrinkage, entry and exit criteria, stepwise tests of significance, ...? Well because, fortunately or unfortunately, there <u>are</u> many situations in which it certainly "do make some nevermind" how you weight the variables in forming the composite.

Pruzek and Frederick (1978) distinguished those situations in which unit weights are as effective as least squares weights from those in which they are not. They identified the <u>factorial complexity</u> of the joint predictor/criterion variable correlation structure as the crucial indicator when discrepant results are produced by the different weighting schemes. Darlington (1978) demonstrated the same point with an index he termed <u>validity concentration</u> of the predictors and criterion. Bentler and Woodward (1979) presented a thorough analytical treatment of

regression on composites, derived from a reconceptualization of the basic linear regression model. Their review of pertinent literature is most complete.

The message from these analytical treatments is quite clear. Both factorial complexity and validity concentration refer to the extent to which the correlation structure among all variables (predictor and criterion) is homogeneous or heterogeneous. The results of these treatments indicate that when the intercorrelations of the variables are of approximately the same size, simplified (unit) weighting schemes will perform as well as optimal (statistically-derived) weighting schemes. When these correlations are of varying size, however, a significant loss in predictive accuracy will result from the use of simplified rather than optimal weights.

Purpose of this Paper

The purpose of the present paper is twofold. First, a demonstration of the effects of simple vs. optimal weighting schemes under conditions which favor each is presented. This is done on simulated data to clearly exemplify the contrast in factorial complexity discussed earlier and the situations in which simplified weighting schemes may and may not be used with equal predictive efficiency. Secondly, the guidelines for weighting in multiple linear regression are extended to other multivariate analysis techniques not discussed in the unit weighting literature to date. The simulated data used in the multiple linear regression (MLR) analysis is also used in extending the guidelines for simplified weighting schemes to canonical correlation (CANCORR) analysis, multivariate analysis of variance (MANOVA) and linear discriminate analysis (LDA).

Relationship Among Multivariate Techniques

All four of the multivariate techniques discussed in this paper are interrelated in that they each operate on a composite of several variables formed according to some pre-established rule--usually the familiar least squares criterion. Harris (1975) has clearly pointed out and shown that once this composite if formed, the analysis of this single variable proceeds as in the familiar univariate methodology. Knapp (1978) has invoked a similar note of generality of these techniques by showing how virtually all parametric statistical tests of significance can be formulated as special cases of the general canonical correlation

framework depicted below in which a set of p predictor variables, X, is related to a set of q criterion variables, Y.

$$X_1, X_2, \ldots, X_p \longleftrightarrow Y_1, Y_2, \ldots, Y_q \qquad (2)$$

The present author suggests that two characteristics of these variable sets are paramount in distinguishing among parametric hypothesis testing techniques. The first is whether the set (X or Y) consists of one variable or more than one variables. The second is whether the set is composed of continuously scaled (numerical) variables or dichotomous, dummy-coded (categorical) variables. Figure 1 displays the sixteen possible combinations of these characteristics of predictors and criteria, along with the hypothesis testing technique utilized in each.

Figure 1 about here

The symmetry of the matrix of techniques is noteworthy. It indicates the relative unimportance of which variable set (X or Y) is termed the predictor set and which is termed the criterion set. The canonical model is thus a truly general significance testing system, as Knapp (1978) shows. It is also a useful conceptual framework in viewing the interrelationships and distinctions among the members of the family of statistical methods, as shown in Figure 1.

In presenting the statistical methods in this way, one can see the rationale for extending the simplified weighting principles, so well documented in the multiple linear regression literature, to other members of its family. The techniques discussed in this paper are all multivariate and involve continuously scaled variables. They are shown along the bottom row (or last column) of Figure 1.

Procedure

To demonstrate the effects of simple weighting schemes on these multivariate techniques under varying (extreme) degrees of factorial complexity among the variables, nine variables were generated with the intercorrelation structure shown in Table 1. The nine variables were simulated to represent two subsets of variables (X_1 - X_4 and X_5 - X_8) which intercorrelate highly among themselves (r = .80), but near zero with each other (r = .10); and an external variable (X_9) which correlates equally with all others (r = .60). Utilizing these nine variables in a variety of combinations of predictor and criterion sets for the multivariate analyses discussed provides ample demonstration of the conditions in which simplified weighting schemes are and are not recommended. All analyses were performed using SPSS, Version 7.2 (Nie, Hull, Jenkins, Steinbrenner & Bent, 1975) and MULTIVARIANCE, Version V, (Finn, 1976).

Table 1 about here

The correlation matrix was first submitted to a common factor analysis to document the degree of factorial complexity among various subsets of the variables. The results are shown in Table 2. The least complexity is shown in the analysis of X_1 through X_4, in which a single common factor emerges, accounting for 85 percent of the variance among the four variables. Obviously, this same result would occur in a factor analysis of X_5 through X_8. A similar degree of low factorial complexity is shown when X_9 is included with X_1 - X_4, although only 78% of the variance among the five variables is accounted for by the

single common factor. In both cases, the homogeneity of the intercorrelations which leads to such factorially simple structure would suggest very little if any loss in predictive accuracy by going to a unit weighting scheme.

Table 2 about here

A higher degree of factorial complexity is represented in the factor analysis of all nine variables and that of X_1-X_8. In both cases, two common factors emerge, accounting for 84.4 and 85 percent of the variables' variance, respectively. By ignoring the dual pattern in interrelationships indicated by the two factor solution, and applying the same weight to all variables in this set, a loss in predictive accuracy is expected.

Results

Multiple Linear Regression (MLR)

Four multiple linear regression analyses were performed to illustrate the guidelines suggested for employing simple weighting schemes. Table 3 contains the standardized least squares weights (b_i), the multiple correlation based upon the optimally weighted composite (R), the (simple) correlation based upon the unit-weighted sum of predictors (r_u) and the simple correlation based upon weights obtained in a post hoc manner by modifying the optimal weights resulting from MLR (r_p).

Table 3 about here

The regressions of variables belonging to a factorially simple system, X_1-X_4 and X_1-X_4, X_9, show identically zero loss in predictive accuracy when unit weights are substituted for optimal weights. A closer look at these two regressions shows this to be not surprising, in that the least squares weights themselves are all equal (b_i = .31, .18, respectively). This is a direct result of the homogeneity among intercorrelations, and defines the condition under which the unit weights result will not differ from the optimal weight result. In the regression of X_1 on X_2, X_3 and X_4, the predictor-criterion correlations are all .80 and the predictor intercorrelations are all .10. In the regression of X_9 on X_1-X_4, the predictor-criterion correlations are all .60 and the predictor intercorrelations are all .10. To the extent that the correlation structure is so uniform, equal weights will suffice.

As the homogeneity of intercorrelation is disturbed, however, the need for differential weighting is apparent. The correlational structures of X_1-X_8 and all nine variables were shown to be more complex in the factor analyses previously presented. The regression of X_1 on X_2-X_8 shows a considerable loss of predictive accuracy when equal weights are employed (r = .4618) from when optimal weights are used (R = .8594). This is again seen in light of the differing predictor-criterion correlations existing in this set (r = .80 for X_2-X_4, r = .10 for X_5-X_8) and differing predictor intercorrelations (r = .80 within X_2-X_4 and X_5-X_8, but .10 across these two subsets). Another example, showing a loss in prediction for the same reason, is in the regression of X_1 on X_5-X_9. The

optimally weighted multiple correlation is .7057, while the unit-weighted composite correlates only .2099 with the criterion, X_1.

In the factorially complex cases, the researcher who first performs an MLR analysis and then seeks to simplify the formulation of the composite based upon the MLR results would fare reasonably well. That is, if simplified weights are chosen as modifications of statistically-derived optimal weights, the prediction would not suffer so drastically. An inspection of the weights in the regression of X_1 on X_2 through X_8 would likely lead the researcher to assign weights of +1 to X_2-X_4 and 0 to X_5-X_8. The resulting simple correlation is .8593 (note that this is equivalent to the first regression in Table 3), a virtually zero loss in predictive accuracy. The same process applied to the last regression (X_1 with X_5-X_9) would probably result in assigning a weight of +1 to X_9 and 0 to X_5-X_8. This leaves the simple correlation between X_1 and X_9, which is .60, representing a far more palatable loss from the optimal multiple R of .7057.

Canonical Correlation (CANCORR)

Three canonical correlations were calculated on combinations of the nine variables to illustrate the effects of factorial complexity on choice of weighting scheme. Although each CANCORR analysis shown could have more than one canonical correlation derived on the variable sets, the data were simulated so as to concentrate all of the interrelationship of the first pair of canonical variates. The results of these analyses are shown in Table 4.

Table 4 about here

The first two CANCORR analyses involve homogeneous correlation structures. The first considers variables X_1-X_4. Within this four variable system all correlations are equal (r = .80). Thus, the optimal standardized weights are equal (b_i = .53) and unit weighting will show no loss from optimal weighting, as in the analogous case in MLR. The second analysis illustrates an additional consideration necessary in CANCORR, which does not exist in MLR. In correlating X_1-X_4 with X_5-X_8, a more factorially complex structure is under investigation, and one might expect the need for differential weights. The results in Table 4 do not indicate this, however. The optimal weights are all equal (b_i = .27), and the canonical correlation retains its value of .1173 whether these or unit weights are employed. The important factor here is that we have the same form of homogeneous correlation within and across predictor and criterion variable sets as before. That is, both sets of variables on each side of the equation have intercorrelations of .80 among themselves, and predictor-criterion correlations of .10. Since CANCORR differs from MLR in that it has multiple variables in both predictor and criterion sets, the factorial complexity is judged relative to two possible composites rather than one.

The third canonical correlation illustrates a greater degree of factorial complexity. The external variable, X_9, is added to the predictor set, X_5-X_8, in relation to the criterion set, X_1-X_4. This adds another stream of correlation to that which existed in the second CANCORR discussed above. The standardized optimal weights are no longer equal, as shown in Table 4, and the canonical correlation of .7654 is reduced to .2277 when unit-weighted composites are substituted. The homogeneity of within and across set correlations in the second CANCORR

(r = .80, .10, respectively) is disrupted by the addition of X_9 which correlates .60 both within its own set (X_5-X_8) and with the criterion set (X_1-X_4) as well.

As in the MLR examples in which unit weighting of all variables shows serious reduction in predictive accuracy, modification of optimal weights provides a useful compromise. Inspection of the optimal weights in the last canonical correlation analysis would probably lead the researcher to assign weights of +1 to X_1-X_4 on the criterion side, +1 to X_9 on the predictor side, and 0 to X_5-X_8. This yields the same set of predictors and criterion as was investigated in MLR analyses--X_9 regressed on X_1-X_4. The results there (c.f., Table 3) indicated no loss in prediction from optimal to unit weighted composites (R = r = .6508). In the CANCORR we contrast this post hoc weighting result with the optimal weighting (R = .7654) and realize little loss in prediction.

MANOVA AND LDA

In order to ilustrate the effects of composite weighting schemes in multivariate analysis of variance (MANOVA) and linear discriminant analysis (LDA), an additional feature of the data needed to be represented--treatment group differences on the nine variables already specified. The present author has noted elsewhere that the MANOVA hypothesis testing venture is a two-dimensional problem (Gabriel, 1979). One dimension concerns itself with the relationships among criterion or dependent variables. For the present study, these have been simulated as shown in Table 1. The second dimension has to do with relationships among treatment groups in terms of these dependent variables.

Table 5 about here

For purposes of this study, four treatment groups were represented ($n_i = 30$), with means and standard deviations of the nine dependent variables as shown in Table 5. The patterns of differences across groups were simulated to represent a simple structure of treatment group differences. Note that variables X_3, X_4, X_7, X_8, X_9 show no differences across the four groups, while the remaining variables (X_1, X_2, X_5, X_6) show the same general pattern of differences--Group 4 the highest, Group 2 the lowest, and Groups 1 and 3 at the mean of each variable. This situation represents a "high concentration of multivariate noncentrality" (Olson, 1974) in MANOVA, resulting in only one discriminant function needed to account for treatment group differences. Olson's (1974) concept of multivariate noncentrality concentration is expressing the same notion as that of factorial complexity (Pruzek and Frederick, 1978) and validity concentration (Darlington, 1978) used previously in this paper.

The MANOVA and LDA results are presented together because the two methods often go hand in hand in testing for group differences on multiple criteria. A distinction can be made between them in terms of the choice of test statistic used. The choice of the MANOVA test statistic itself is the subject of some controversy in the methodological literature (Harris, 1975; Olson, 1976; Stevens, 1979). The various test statistics differ primarily in the way in which they aggregate the multivariate differences represented by the multiple roots (eigenvalues) of the characteristic equation. Hotelling's trace criterion (1951) adds them up; Wilks' lambda (1932) is a multiplicative function of them; Roy's largest root criterion (1957) considers only the largest eigenvalue in its test. Harris (1975) suggests that correspondence between the overall

MANOVA and the leading discriminant function demands the congruence of Roy's largest root test for both, although he acknowledges Wilks' lambda (or its approximation) to be the most frequently employed test statistic. The current paper is not intended to shed any light on these issues. In demonstrating the effects of various weighting schemes in MANOVA, it is seen as most important to represent applied research practice. The multivariate F statistics reported in the examples to follow are those obtained via Rao's (1952) transformation of Wilks' lambda, whose sampling characteristics closely approximate the F distribution.

Another word of caution before proceeding to the MANOVA and LDA results. Comparisons of results from different composite weighting schemes are presented in terms of hypothesis test statistics and significance testing results, unlike those for MLR and CANCORR. These are not directly comparable, in that they tap different sampling distributions which arise from different statistical models and assumptions. They are presented here simply as indicators of the discrepancies which arise from the different approaches.

Four MANOVAs and corresponding LDAs were conducted on the simulated data. Table 6 contains the results of these, including the univariate F statistic of group differences for each of the nine variables, the standardized discriminant function weights for each of the five dependent variable sets investigated, and the multivariate F statistic, degrees of freedom, and p value for these sets. Also shown in Table 6 is the univariate F and p value for the unit-weighted composite (F_u) and the univariate F and p value for the composite formed in a post hoc manner (F_p), that is, by interpreting the results of the LDA and choosing

weights on this basis. This post hoc specification has implications for the appropriate hypothesis test and, as discussed above, the p-value presented should not be interpreted literally.

Table 6 about here

The multivariate F test on X_1-X_4 is highly statistically significant ($F = 5.43$, $p < .0001$) when an optimal weighting scheme is employed. The weights derived show a clear contrast between the four variables. X_1 and X_2 are highly positively weighted, while X_3 and X_4 are highly negatively weighted. Recall that X_1-X_4 are all intercorrelated .80--a homogeneous correlation structure. MLR results using only these four variables indicated that equal weighting would suffice. Here, however, the four are relating to the criterion (treatment group differences) quite differently. Variables X_1 and X_2 show the same degree of treatment group difference (univariate $F = 5.00$), and are thus weighted equally ($b_i = 1.15$). Variables X_3 and X_4 also relate to the criterion equally (univariate $F = 0.00$) and are weighted the same ($b_i = 1.02$). The weights differ across these two pairs, however, because they are different from each other in their relationship with the criterion. The highly negative weights on X_3 and X_4 are evidence of their strong suppressor influence in the multivariate hypothesis test, and they should be retained in a simplified composite. To simply sum the four variables would clearly serve only to diminish the potency of X_1 and X_2 as positive discriminators. This is indicated by the large drop in the test of mean differences when unit weighted (all +1) composite is employed ($F_u = 1.47$, $p < .2263$). If the appropriate sign--determined through LDA--is retained in a unit

weighting, and the composite is the difference between X_1, X_2 and X_3, X_4, the test of group differences still indicates large differences ($F_p = 25.00$, $p < .0001$).

A similar result is shown in the MANOVA on X_5-X_8. The exact magnitude of the multivariate F and the subsequent drop when a unit weighted composite is substituted differs somewhat from that of the X_1-X_4 analyses only because the magnitude of group differences is not the same. The pattern is the same, however. The discriminant function weights describe a contrast of pairs of variables, so that a unit-weighted sum ($F = .83$, $p < .4815$) will not adequately maintain the discriminating power of the optimally weighted composite ($F = 3.88$, $p < .0001$). Inspection of the LDA results suggests a composite representing the contrast between X_5 and X_7, X_8. Again, when statistically-derived results are considered first, a simplified composite shows only a slight reduction in detecting group differences ($F_p = 3.57$, $p < .0163$).

MANOVA on X_1-X_8 and X_1-X_9 show the same mix of positive and negative weights among the variables in discrminating among treatment groups. Unit weighted composites evidence a large drop from the optimally weighted results, while simplified composites based upon LDA results again seem palatable compromises.

SUMMARY

Three procedures for weighting variables in a composite were examined for four multivariate analysis techniques. Guidelines from the literature on multiple linear regression for the use of simple unit weights were clarified and exemplified on a simple, simulated data set. When these guidelines are extended to the more complex multivariate techniques (with multiple criteria as well as multiple predictors), they involve correspondingly more complex considerations, but the principles which formulate the guidelines remain the same. The issue is always one of the homogeneity of the joint correlation structure of predictor and criterion variables.

A simplified weighting process, based upon the optimal weights, but employing the unit weights dictated by the analysis, invariably indicated encouraging results. By availing oneself of the precision of least squares results, but simplifying to unit weighting (-1, 0, +1), the interpretability as well as generalizability of the composite variable itself is enhanced.

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Figure 1

Classification of Parametric Hypothesis Testing Techniques

$X_1, X_2, \ldots, X_p \longleftrightarrow Y_1, Y_2, \ldots, Y_q$

		CRITERIA (Y_1)			
		ONE		MANY	
		CAT.	NUM.	CAT.	NUM.
PREDICTORS (X_1) ONE	CATEGORICAL	X^2	t	X^2	LDA
	NUMERICAL	t	r	ANOVA	MLR
MANY	CATEGORICAL	X^2	ANOVA	X^2	MANOVA
	NUMERICAL	LDA	MLR	MANOVA	CANCORR

Table 1

Variable Intercorrelation Matrix

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
X_1	1.00	.80	.80	.80					.60
X_2		1.00	.80	.80					.60
X_3			1.00	.80			(all .10)		.60
X_4				1.00					.60
X_5					1.00	.80	.80	.80	.60
X_6						1.00	.80	.80	.60
X_7							1.00	.80	.60
X_8								1.00	.60
X_9									1.00

Table 2

Factor Structure of Variable Sets

Variable Sets	X_1-X_4	X1-X4,X9	X1-X8		X1-X9	
Factors	I	I	I	II	I	II
Variables						
X_1	.89	.89	.67	.59	.89	.06
X_2	.89	.89	.67	.59	.89	.06
X_3	.89	.89	.67	.59	.89	.06
X_4	.89	.89	.67	.59	.89	.06
X_5			.67	-.59	.06	.89
X_6			.67	-.59	.06	.89
X_7			.67	-.59	.06	.89
X_8			.67	-.59	.06	.89
X_9		.67			.63	.63
Percent Variance Accounted For	.85%	77.9%	85%		84.4%	

Table 3

Multiple Linear Regression on Variable Sets

Variable Sets...	$X_1 \leftarrow (X_2\text{-}X_4)$	$X_9 \leftarrow (X_1\text{-}X_4)$	$X_1 \leftarrow (X_2\text{-}X_8)$	$X_1 \leftarrow (X_5\text{-}X_9)$
Variables	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i
X_1	(Criterion)	.18	(Criterion)	(Criterion)
X_2	.31	.18	.31	
X_3	.31	.18	.31	
X_4	.31	.18	.31	
X_5			.002	-.13
X_6			.002	-.13
X_7			.002	-.13
X_8			.002	-.13
X_9		(Criterion)		.92
Multiple R (Optimal Weights)	.8593	.6508	.8594	.7057
Simple r_u (Unit Weights)	.8593	.6508	.4618	.2099
Simple r_p (Post hoc Weights)	--	--	.8593	.6000

Table 4

Canonical Correlation on Variable Sets

Variable Sets...	$(X_1, X_2) \leftrightarrow (X_3, X_4)$	$(X_1\text{-}X_4) \leftrightarrow (X_5\text{-}X_8)$	$(X_1\text{-}X_4) \leftrightarrow (X_5\text{-}X_9)$
Variables	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i
X_1	.53	.27	.27
X_2	.53	.27	.27
X_3	.53	.27	.27
X_4	.53	.27	.27
X_5		.27	-.19
X_6		.27	-.19
X_7		.27	-.19
X_8		.27	-.19
X_9			1.30
Canonical R (Optimal Weights)	.8889	.1173	.7654
Simple r_u (Unit Weights)	.8889	.1173	.2277
Simple r_p (Post hoc Weights)	--	--	.6508

Table 5

Group Means ($\bar{X}$) and Standard Deviations (s)
on All Variables

	Group 1		Group 2		Group 3		Group 4	
	$\bar{X}$	s	$\bar{X}$	s	$\bar{X}$	s	$\bar{X}$	s
X_1	20	8	16	8	20	8	24	8
X_2	20	8	16	8	20	8	24	8
X_3	20	8	20	8	20	8	20	8
X_4	20	8	20	8	20	8	20	8
X_5	40	8	36	8	40	8	44	8
X_6	40	16	36	16	40	16	44	16
X_7	40	8	40	8	40	8	40	8
X_8	40	16	40	16	40	16	40	16
X_9	50	10	50	10	50	10	50	10

Table 6

MANOVA and LDA on Variable Sets

Variable Sets		(X_1-X_4)	(X_5-X_8)	(X_1-X_8)	(X_1-X_9)	(X_1-X_2)
Variables	Std. Wt., Univ. F	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i	Std. Wt., b_i
X_1	5.00	1.15		.89	.94	.54
X_2	5.00	1.15		.89	.94	.54
X_3	.00	-1.02		-.79	-.63	
X_4	.00	-1.02		-.79	-.63	
X_5	5.00		1.70	1.08	1.13	
X_6	1.25		.39	.24	.34	
X_7	.00		-.93	-.60	-.45	
X_8	.00		-.93	-.60	-.45	
X_9	.00				-.72	
Mult. F (Optimal Weights)		5.43	3.88	3.97	3.88	2.66
df		12,199	12,199	24,317	27,316	6,230
p value		.0001	.0001	.0001	.0001	.0163
Univ. F_u (Unit Weights)		1.47	.83	2.01	1.49	5.56
df		3,116	3,116	3,116	3,116	3,116
p value		.2263	.4815	.1157	.2199	.0014
Univ. F_p (Post hoc Weights)		25.00	3.57	20.45	11.25	5.56
df		3,116	3,116	3,116	3,116	3,116
p value		.0001	.0163	.0001	.0001	.0014